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elastic bodies; Coulomb friction; dynamic stress intensity factor; quasi-static fracture of power-law inhomogeneous viscoelastic material.

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

This report summarizes progress on several theoretical questions concerning contact and fracture phenomena for viscoelastic material. Specifically, the following topics are discussed: (1) the dynamic, steady-state propagation of a semi-infinite crack in a general, isotropic and homogeneous linearly viscoelastic layer of finite thickness; (2) the determination of the angular dependence of the stress field in the neighborhood of a dynamically propagating crack tip in linearly viscoelastic material; (3) the quasi-static and steadystate sliding of a rigid indentor over a power-law inhomogeneous, linearly viscoelastic halfplane; and (4) the quasi-static propagation of a semi-infinite, mode I crack in a power inhomogeneous, linearly viscoelastic body. ELECTE

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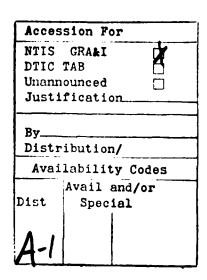
### FRACTURE PROBLEMS IN POWER LAW VISCOELASTIC MATERIALS

FINAL SCIENTIFIC REPORT

GRANT AFOSR-82-0152

1 APRIL 1983 - 31 MARCH 1984

Professor Jay R. Walton Department of Mathematics Texas A&M University College Station TX 77843





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### SECTION 1. Introduction.

This report is a summary of the research conducted by the principal investigator, Jay R. Walton, under AFOSR Grant 82-0152 for the period 4/1/82 - 3/31/84. The subject areas of the research involved dynamic and quasi-static analyses of fracture and contact phenomena for homogeneous and inhomogeneous linearly viscoelastic material. The particular problems studied were:

- the dynamic, steady-state propagation of a semi-infinite crack in a general, isotropic and homogeneous linearly viscoelastic layer of finite thickness;
- 2. the determination of the angular dependence of the stress field in the neighborhood of a dynamically propagating crack tip in linearly viscoelastic material;
- the quasi-static and steady-state sliding of a rigid indentor over a power-law inhomogeneous, linearly viscoelastic half-plane;
- 4. the quasi-static propagation of a semi-infinite, mode I crack in a power-law inhomogeneous, linearly viscoelastic body.

A description of the progress obtained on each of these problems is contained in the next section. The final section of this report is a list of the papers written under this grant (published, submitted for publication or in preparation).

# SECTION 2. Summary of Research Accomplishments.

1. As part of a long term collaborative research effort with Professor R. A. Schapery of the Departments of Aerospace and Civil Engineering of Texas A&M University, the study of the dynamic fracture of viscoelastic material was continued. The first problem considered was a dynamic analysis of a steadily propagating, semi-infinite, anti-plane strain (mode I) crack in an infinite, homogeneous and isotropic, linearly viscoelastic body. This work is contained in the parer: "On the steady-state propagation of an anti-plane shear crack in an infinite general linearly viscoelastic body," by J. R. Walton, Quart. Appl. Math., April 1982. One of the principal results in the paper is the derivation of a simple closed form expression for the stress intensity factor and, indeed for the entire stress distribution in the body. This stress field was

shown to have a universal dependence upon the shear modulus and crack speed from which pertinant qualitative and quantitative information is easily obtained.

Suvsequently, the corresponding provlem for a viscoelastic layer of finite thickness was considered. By similar methods to those employed in the infinite body case, the stress intensity factor for this much more complicated problem was calculated. An infinite series representation for this stress intensity factor was derived, each term of which can be calculated recursively in closed form, from which a simple universal dependance upon crack speed and material properties is revealed. The work is contained in the paper, "The dynamic, steady-state propagation of an anti-plane shear crack in a general linearly viscoelastic layer," which has been submitted for publication in the Journal of Applied Mechanics.

2. The principal focus of the above study was the construction of convenient expressions for the dynamic stress intensity factor. Motivated by discussions at the International Workshop on Dynamic Fracture held at the California Institute of Technology in February of 1983, the problem of determining the angular dependence of the stress field in the neighborhood of a dynamically propagating crack was considered. Such calculations have been done for cracks in elastic material but not in viscoelastic material. For an elastic analysis, it is convenient to perform the calculation by considering the equations of motion in local polar coordinates at the crack tip, thereby ignoring any far field complications. However, such is not the case for a viscoelastic analysis owing to the presence of convolution integrals in the governing differential equations to account for the entire stress and strain history. The approach adopted was to work with a general Fourier integral representation of the stress field constructed as the solution to a particular boundary value problem. Consequently, the geometry and loading for the specific problem are considered. An asymptotic expansion of the stress in local polar coordinates about the crack tip is then derived from the global Fourier representation. In this way it was shown first that for the dynamic, steady-state propagation of a semi-infinite mode III crack through an infinite, isotropic and homogeneous, general linearly

viscoelastic body, the near tip shear stress is asymptotic to  $f(V,\theta)K/r^{\frac{1}{2}}$ , where  $(r,\theta)$  are local polar coordinates at the crack tip, K is the dynamic stress intensity factor (already determined in a previous paper) and  $f(V,\theta)$  is a universal function of  $\theta$  and the crack speed, V. Indeed, f is the same function that occurs in the corresponding elastic analysis with a shear modulus equal to the glassy viscoelastic modulus. These results are reported in the paper "Dynamic steady-state fracture propagation in general linear viscoelastic material," published in the conference proceedings of the Workshop on Dynamic Fracture, Ed. W. G. Knauss, K. Ravi-Chandar, A. J. Rosakis, California Institute of Technology, 1983.

Subsequently, the same calculation was carried out for the more complicated problem of a mode III crack in a layer of finite thickness. It was shown that the field has the smae form as for an infinite body, only the dynamic stress intensity factors being different.

Currently, a similar analysis is being undertaken for the mode I crack. Once the investigation of this case is completed, all of these results will be collected in a paper to be submitted for publication.

An investigation of contact and fracture problems for inhomogeneous linearly viscoelastic material was initiated. The first problem studied was a frictional contact problem for power-law inhomogeneous material. Specifically, a simple closed form solution was derived for the quasistatic sliding with Coulomb friction of a rigid indentor over a linearly viscoelastic, half-plane for which Poisson's ratio is constant but for which the shear modulus has a power-law dependence upon time and upon depth from the half-plane surface. Such a modulus models a material that stiffens with depth, as might occur, for instance, with the frictional interaction of a rigid asperity sliding over a polymeric or rubbery material when the heat generated produces a softening of the material near the body's surface. Of principal interest in this study was the determination of the combined effects of material viscoelasticity and inhomogeneity upon the friction coefficient. It was demonstrated that the friction coefficient has a simple functional dependence upon the model parameters and that its magnitude is greatly affected by the material inhomogeneity..

This work is contained in the paper, "The sliding with Coulomb friction of a rigid indentor over a power-law inhomogeneous linearly viscoelastic half-plane," J. Appl. Mech. Vol. 106, 1984, pp. 289-293.

4. Subsequent to the completion of the friction analysis described in 3., an investigation of the fracture of power-law inhomogeneous material was initiated. The first problem considered was the quasi-static, steady-state problem of a semi-infinite mode I (opening mode) crack propagating through an infinite, power-law inhomogeneous, isotropic linearly viscoelastic body. The power-law spatial variation of the shear modulus is from the crack plane into the body. The initial treatment assumed also a power-law dependence in time for the shear modulus. However, by employing a combination of the techniques used in the dynamic analysis for crack propagation in homogeneous material and those in the quasi-static friction work for inhomogeneous material, the restriction to power-law time behavior was removed and the problem for a more general time dependence was solved. The shear modulus is assumed to be monotone decreasing and convex as a function of time. This work is nearly completed and will soon be submitted for publication.

# SECTION 3. Papers Resulting form Grant Supported Research.

- 1. "The dynamic, steady-state propagation of an anti-plane shear crack in a general linearly viscoelastic layer," submitted to J. Appl. Mech.
- 2. "The sliding with Coulomb friction of a rigid indentor over a power-law inhomogeneous linearly viscoelastic half-plane," J. Appl. Mech. (106) 1084, 289-293.
- 3. "The steady-state propagation of a semi-infinite, mode I crack in an infinite power-law inhomogeneous, linearly viscoelastic body," in preparation.

### Invited Presentation.

 "Dynamic steady-state fracture propagation in general linear viscoelastic material," <u>Workshop on Dynamic Fracture</u>, W. G. Knauss, K. Ravi-Chandar, A. J. Rosakis, Ed's, Feb. 1983, sponsored by the Army Research Office and the National Science Foundation. FINAL SCIENTIFIC REPORT of work completed during the grant years 4/1/82 to 3/31/84 under AFOSR Grant No. 82-0152

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